EXTRACTION OF ROOTS BY LOGARITHMS.

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LOGARITHS would afford the readiest method of computing the roots of numbers if we had Tables of sufficient extent.

Let it be required to find the n^{th} root of a.

1.—Put $r + x = \sqrt[n]{a}$, then $\log r = \frac{1}{n} \log a$, and r is the number whose logarithm is $\frac{1}{n}$ of the logarithm of a. Common logarithms may be used in this crse.

2.—Put $r + x = \sqrt[n]{a}$, where r is the integral part of the root; then, using Napierian logarithms,

$$\log (r + x) = \frac{1}{n} \log a.$$

Expanding $\log (r + x)$ and transposing $\log r$,

$$\left(\frac{x}{r}\right) - \frac{1}{2}\left(\frac{x}{r}\right)^2 + \frac{1}{3}\left(\frac{x}{r}\right)^3 - \frac{1}{4}\left(\frac{x}{r}\right)^4 + \text{\&c.} = \frac{1}{n} \log a - \log r \ldots (1)$$

Reverting this series,

$$\frac{x}{r} = \left(\frac{1}{n} \log a - \log r\right) + \frac{\left(\frac{1}{n} \log a - \log r\right)^2}{1\,.\,2} + \frac{\left(\frac{1}{n} \log a - \log r\right)^3}{1\,.\,2\,.\,3} + \frac{\left(\frac{1}{n} \log a - \log r\right)^4}{1\,.\,2\,.\,3\,.\,4} + \frac{\left(\frac{1}{n} \log a - \log r\right)^5}{1\,.\,2\,.\,3\,.\,4\,.\,5} + \text{\&c}$$

Example. — Required the one-hundredth root of 2.

Put $a = 2$, $r = 1$ and $n = 100$; then

$$\sqrt[100]{2} = 1 + \frac{\log 2}{100} + \frac{(\log 2)^2}{1\,.\,2\,.(100)^2} + \frac{(\log 2)^3}{1\,.\,2\,.\,3\,.(100)^3} + \frac{(\log 2)^4}{1\,.\,2\,.\,3\,.\,4\,.(100)^4} + \text{\&c.}$$

The Napierian logarithm of 2 is 0.693147180559 +,

$$\therefore \sqrt[100]{2} = 1.0069555499 + .$$